equation and two components of the vector vorticity equation (the steady Helmholtz equations[3]), although Refs. 1 and 2 have the latter two somewhat disguised, and not identified. Indeed, it would be surprising if the flow problem they attacked were described by anything other than the steady Helmholtz equations. It is, after all, steady, rotational, inviscid, incompressible flow and the authors eliminate the pressure by cross-differentiation.

One can show the identity of equations in Refs. 1 and 2 with the θ and z Helmholtz equations. In fact, it was this commenter's atempt to do this when Ref. 1 appeared which led to his pointing out to Prof. Hamed the erroneous third equation [Eq. (7) and Eq. (A10) of Ref. 1] which has been corrected in Ref. 2.

Let u,v,w be the velocity components in the r, θ, z directions, respectively, of a cylindrical coordinate system. The corresponding components of the vorticity (the curl of the velocity vector) are:

$$\eta = \frac{1}{r}\frac{\partial w}{\partial \theta} - \frac{\partial v}{\partial z}, \; \xi = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}, \; \zeta = \frac{\partial v}{\partial r} + \frac{v}{r} - \frac{1}{r}\frac{\partial u}{\partial \theta} \quad (1)$$

ξ is defined in Ref. 1, Eq. (5) and ζ in Ref. 2. They do not use η explicitly.

The continuation equation [Eq. (4) of Ref. 1] is

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

With the definition equations (1), Eq. (6) of Ref. 1 is easily seen to be

$$u\frac{\partial \xi}{\partial r} + \frac{v}{r}\frac{\partial \xi}{\partial \theta} + w\frac{\partial \xi}{\partial z} = \eta\frac{\partial v}{\partial r} + \frac{\xi}{r}\frac{\partial v}{\partial \theta} + \zeta\frac{\partial v}{\partial z} + \frac{u\xi}{r} - \frac{v\eta}{r} \quad (3)$$

which is the θ Helmholtz equation. To convert the equation of Ref. 2 to the z Helmholtz equation is somewhat harder. One first replaces the first parenthesis on the right-hand side by the right-hand side of Eq. (3) of this Comment. Then, the first, third, and fourth terms on the right-hand side of the Ref. 2 equation combine to yield only $w\zeta\partial v/\partial z$, since the rest of these terms either cancel or vanish by continuity, Eq. (2) of this Comment. This remaining term cancels a term on the left, and the $u\partial v/\partial r$ terms on both sides cancel. Finally, if $v\zeta\partial w/\partial z$ is added to both sides, and the continuity equation is applied again, we find

$$\eta\frac{\partial w}{\partial r} + \frac{\xi}{r}\frac{\partial w}{\partial \theta} + \zeta\frac{\partial \omega}{\partial z} = u\frac{\partial \zeta}{\partial r} + \frac{v}{r}\frac{\partial \zeta}{\partial \theta} + w\frac{\partial \zeta}{\partial z} \quad (4)$$

which is the z Helmholtz equation.

The equations solved in Ref. 1 may thus be reduced to Eqs. (2-4) herein, the continuity equation, and the θ and z Helmholtz equations.

There is no contention here that there is anything wrong with the numerical method or the results of Ref. 1. The purpose of this Comment is only to clarify the basic equations used there, and show that they are the equations one should expect for the problem, and not some novel set of relations, as might first appear to be the case.

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Equivalent *G/E* of Helicopter Rotor Blades

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Introduction

RECENTLY the concept of equivalent G/E has been used in order to describe the structural behavior of helicopter rotor blades.[1,2] As stated in those references, and as agreed for many years, it is possible to treat helicopter blades as beams by considering equivalent beam properties which are obtained by evaluating certain integrals over the cross section, hence averaging the effects of different materials presented in the cross section. Hodges[1,2] claims that averaging leads to effective values of G/E which may differ from those encountered in isotropic structures by one or more orders of magnitude.

The purpose of the present Note is to show, based on existing helicopter blades, that this last statement is in error and, in fact, the typical equivalent G/E of modern helicopter blades is very similar to that of isotropic materials and is not by any means different by one or more orders of magnitude.

Calculations of the Equivalent *G/E*

The best way of finding the equivalent G/E of blades is using the equivalent beam properties of the blade, which are validated by experiments. The properties that are functions of the equivalent E are the blade tension stiffness (EA) and the bending stiffness of the blade about the two principal axes (EI_f), (EI_t), which are the flapwise and edgewise stiffness, respectively. The most important property that is a function of G is the torsional stiffness (GJ). As in Ref. 1, a solid ellipse will be chosen as representative of the blade cross section. If the length of the large and small principal axes equal $2a$ and $2b$, respectively, then:

$$(EA) = \pi ab E_{eq} \quad (1a)$$

$$(EI_f) = (\pi/4)ab^3 E_{eq} \quad (1b)$$

$$(EI_t) = (\pi/4)a^3 b E_{eq} \quad (1c)$$

$$(GJ) = [\pi a^3 b^3/(a^2+b^2)]G_{eq} \quad (1d)$$

E_{eq} and G_{eq}, which appear on the right-hand side of Eqs. (1a-d), are the equivalent values that are the subject of this Note. Since the values of the terms on the left-hand side of Eqs. (1a-d) are known, it is possible to obtain the equivalent G/E from these equations, as:

$$\left(\frac{G}{E}\right)_{eq} = \left(1+\frac{b^2}{a^2}\right)\frac{(GJ)}{b^2(EA)} \quad (2a)$$

$$\left(\frac{G}{E}\right)_{eq} = \left(1+\frac{b^2}{a^2}\right)\frac{(GJ)}{4(EI_f)} \quad (2b)$$

It is clear from Eqs. (2) that since b/a varies between 1 and 0 at most, its exact value is not of prime importance when the magnitude of the equivalent G/E is of interest. The value of $(b/a)^2$ is obtained from Eqs. (1b) and (1c):

$$(b/a)^2 = (EI_f)/(EI_t) \quad (3)$$

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Table 1 Results for different helicopter blades

Example No.	Source (Ref. No.)	(EI_f), N-m^2	(EI_t), N-m^2	(GJ), N-m^2	(EA), N	b, m	b/a, Eq. (3)	$(G/E)_{eq}$, Eq. (2a)	$(G/E)_{eq}$, Eq. (2b)
1	3	126.3	3444	149.4			0.192		0.307
2	4	57.1	1813	66.0			0.177		0.298
3	5	0.0461×10^5	1.53×10^5	0.0717×10^5			0.174		0.401
4	6	0.861×10^5	43.95×10^5	0.708×10^5			0.140		0.209
5	7	0.155×10^5	6.641×10^5	0.201×10^5	7.928×10^7	0.0177	0.153	0.809	0.332

Table 1 contains details about the properties of different blades of modern rotors. Since the properties vary along the blade the numbers in the table are typical values that represent the mean value along main portions of the blade. Example 1 is a Mach scaled model of the UTTAS YUH-61A of fiberglass composite construction. The second example is also a model for wind tunnel experiments. In this case the blade construction consists of a basically oval titanium spar, Nomex core in the trailing portion, and foam filler in the nose to maintain contour. Fiberglass cloth was bonded around the Nomex, spar, and filler to form a closed blade skin. The blade of example 3 is of a tail rotor that is made of composite materials. The example 4 blade is a common all-metal blade. The last blade (example 5) belongs to the Advanced Technology Rotor System. In this case the blade employs titanium spar construction with a fiberglass skin and utilizes graphite composite trailing edge strips. The five blades represent different construction techniques, different materials, and different manufacturers. (EA) was given only for the last example and therefore only in that case was the equivalent G/E calculated using Eq. (2a). In all cases the equivalent G/E was calculated using Eq. (2b). If one recalls that for isotropic material $G/E=0.38$-0.4, then it may be concluded that the equivalent G/E of anisotropic blades is very similar to that of isotropic materials and is clearly of the same order of magnitude. None of the cases is even close to the value of 0.025 that was used in Ref. 1 as representative for helicopter blades. It should also be noted that the values of the equivalent G/E according to the two equations in example 5 are considerably different, although they are both of the same order of magnitude. This difference probably results from the approximation where a solid elliptical cross section represents the blade cross section.

Conclusions

It has been shown that the equivalent G/Es of rotor blades—taking into account different kinds of constructions, different materials, and different manufacturers—are very similar to those of isotropic materials. Therefore, many results that were obtained for isotropic beams may also be applied to rotor blades.

There are certain limitations to using the concept of equivalent G/E for investigating the behavior of rotor blades, and it seems preferable to treat the blades as beams by considering equivalent beam properties which may be validated by experiments.

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Reply by Author to A. Rosen

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Dr. Rosen's paper focuses on effective values of G/E for helicopter rotor blades. I have previously stated that effective values of G/E "*may* be much less than unity"[1] and "*may* differ from those encountered in isotropic structures by one or more orders of magnitude."[2] References 1 and 2 do not suggest "using the concept of equivalent G/E for investigating the behavior of rotor blades" as Rosen claims. Its use was limited in Refs. 1 and 2 to assessing the importance of certain terms in the blade torsion equation. In Ref. 2 it is further stated that "the terms in question are not negligible for certain composite rotor blades, especially the flexbeam portion of bearingless rotor blades."

Actual values of G/E for some composite materials with uniaxial fiber orientation are considerably less than 0.4. The value of 0.025 used in example calculations in Refs. 1 and 2 is typical of such materials. These materials are relevant for construction of the flexbeam portion of bearingless rotor blades.

Rosen has chosen several example blade sections to demonstate that equivalent G/E is on the order of 0.4. All these blade sections have torsion rigidity levels that are typical for rotor blade applications. His five examples include nonisotropic metal and composite construction. However, when composite materials are used in blade applications, they normally have fiber orientations that produce a relatively high torsion stiffness. Therefore it is not surprising that his calculated equivalent G/E values are typical of isotropic materials. The important point quoted above from Ref. 2 concerning the flexbeam portion of bearingless rotor blades is not addressed by Rosen. Although the terms in question in the torsion equation are not particularly relevant for portions of blades with relatively high torsion rigidity, they are important

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